

$K = (v/U_\infty^2) \cdot (dU_\infty/dx)$  is the acceleration parameter ( $K_{Cr} = 2.5 \cdot 10^{-6}$ );  $q$  is the thermal flux;  $T_{wa}$  is the temperature of the surrounded surface;  $U_\infty$  is the velocity of the incoming flow;  $n$  is the number of nodes of the computational grid in the direction of the  $y$ -axis;  $M$  is the Mach number;  $Re_x$  is the Reynolds number over the  $x$ -coordinate; and  $C_f$  denotes the local friction coefficient.

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#### HEAT TRANSFER BY MIXED CONVECTION IN A MOVING ROD BUNDLE

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A mathematical model of mixed convection in moving rod bundles is proposed, and cooling in an open space is analyzed. Estimates of the local Nusselt numbers are obtained for quasistabilized conditions.

The formation of fibers from polymer melts is fundamental to the production of synthetic fibers. The physical properties and quality of the fibers depend significantly on the heat-transfer intensity of the moving polymer jets with the surrounding medium. The extension of single fibers has now been fairly completely studied, and the corresponding mathematical models have been constructed [1, 2]. However, it is more common to form bundles of fibers. In such conditions, the spatial interaction of individual fibers and of the whole bundle with the external medium must be considered. The few studies of heat transfer in such conditions have mainly been qualitative and experimental in character [3, 4]. Accordingly, it is necessary to develop mathematical models and methods of calculation of heat transfer in moving fiber bundles. In addition, it is of interest to determine the role of various physical factors in the overall pattern of heat transfer of the fiber bundle with the external medium. Thus, in the case of low velocities and high melt temperature (glass-fiber), free convection is important. Increase in the rate of fiber formation leads to increase in the role of induced convection. It is known [1] that, in the existing conditions of formation of a single synthetic fiber, the proportion of free convection in the overall heat-transfer process is around 10%. As yet, there are no such estimates for bundles of fibers.

The filtrational flow model is widely used for the description of heat transfer in complex rod systems [5, 6]. It reflects the hydrodynamic and thermal interaction of the rods with one another and of the bundle as a whole with the surrounding atmosphere. In the literature, attention focuses on power units with high gas velocities, and accordingly

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dimensionless relations are used for the dynamic and thermal sources. A significant difference between that approach and the present formulation of the problem is the analytical determination of the friction and heat transfer in the volume of the bundle and the matching of the flow in the rod system with the surrounding medium. In contrast to [7, 8], where a flow model with induced convection was constructed, mixed convection is considered in the present work; the results of numerical calculations of some methods of bundle cooling are presented, and expressions for the local Nusselt numbers for rods in quasisteady flow conditions are found.

Basic Equations and Boundary Conditions. Consider the laminar heat transfer of a vertical rod bundle moving in open space (Fig. 1). The basic equations are the equations of generalized filtrational motion within the framework of the boundary-layer model and the Bousinesq model, in the form [9]

$$\begin{aligned} \varepsilon^{-1} \left( u_1 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial r} \right) &= -f + \delta \varepsilon g \frac{T_1 - T_\infty}{T_\infty} + \nu \left( \frac{\partial^2 u_1}{\partial r^2} + \frac{\partial u_1}{r \partial r} \right), \\ \frac{\partial (ru_1)}{\partial x} + \frac{\partial (rv_1)}{\partial r} &= 0, \\ \varepsilon^{-1} \rho c_p \left( u_1 \frac{\partial T_1}{\partial x} + v_1 \frac{\partial T_1}{\partial r} \right) &= \varepsilon^{-1} q + \lambda \left( \frac{\partial^2 T_1}{\partial r^2} + \frac{\partial T_1}{r \partial r} \right). \end{aligned} \quad (1)$$

In deriving these equations, it is assumed that the transverse velocities are considerably less than the longitudinal velocities, the flow is steady, and  $p = p_\infty = \text{const}$ ,  $T_\infty = \text{const}$ . For free convection in the opposite direction,  $\delta = -1$ . A region of moving homogeneous fluid is adjacent to the bundle. Since the flow in the region of the bundle is considered within the boundary-layer framework, it is natural to use the well-known boundary-layer equations also in the external region; they differ from Eq. (1) in that  $\varepsilon = 1$ ,  $f = q = 0$  (in the external region, the subscript 1 is replaced by 2).

Matching of the regions is by means of the following boundary conditions

$$\begin{aligned} u_2(x, R_b) &= \varepsilon^{-1} u_1(x, R_b), \quad v_1(x, R_b) = v_2(x, R_b), \quad T_1(x, R_b) = T_2(x, R_b), \\ \mu \frac{\partial u_2}{\partial r} \Big|_{r=R_b} &= \mu \frac{\partial u_1}{\partial r} \Big|_{r=R_b}, \quad \lambda \frac{\partial T_2}{\partial r} \Big|_{r=R_b} = \varepsilon \lambda \frac{\partial T_1}{\partial r} \Big|_{r=R_b}. \end{aligned} \quad (2)$$

Conditions at infinity and at the beam axis must be added here.

$$\begin{aligned} v_1 = 0, \quad \frac{\partial u_1}{\partial r} = 0, \quad \frac{\partial T_1}{\partial r} = 0 \quad (r = 0), \\ u_2 \rightarrow 0, \quad T_2 \rightarrow T_\infty \quad (r \rightarrow \infty). \end{aligned} \quad (3)$$

To determine  $f$  and  $q$ , the free-cell model for two-phase systems is used [10]. For the case of longitudinal flow around a cylinder forming part of an assembly, the cell is the region between two coaxial cylinders. The inner cylinder is a body of radius  $R_r$  which is moving or in a flow, and the outer cylinder is a fluid shell of radius  $R_\Delta$ ; the relation between  $R_\Delta$  and the porosity of the bundle is given by the expression:  $\varepsilon = 1 - (R_r/R_\Delta)^2$ . In [7-9], a modification of the cell model was proposed: at the outer boundary of the cell,

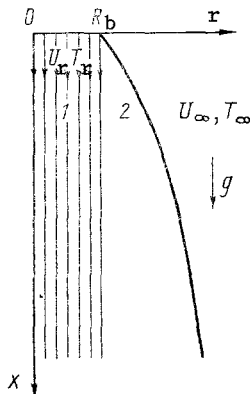


Fig. 1. Cooling of a moving rod bundle in open space.

the conditions for the velocity and temperature of the moving medium are:  $u = U_\Delta$ ,  $T = T_\Delta$  (at the rod surface,  $u = U_r$ ,  $T = T_r$  when  $r = R_r$ ). The arbitrary quantities  $U_\Delta$ ,  $T_\Delta$  are determined by means of the filtrational parameters of the flow. In this interpretation, the individual cells interact with one another in the bundle and are related to the external medium by means of the boundary conditions in Eq. (2). Assuming axisymmetric mixed-convective flow in the cell and using only the zero approximation of the successive-approximation method to solve the boundary-layer equations [11], the following solution in the cell may be obtained:

$$\begin{aligned}
 u = U_r + (U_\Delta - U_r) & \frac{\ln \frac{r}{R_r}}{\ln \frac{R_\Delta}{R_r}} - \delta \frac{U_r (Gr_r/Re_r)}{4 \ln \frac{R_\Delta}{R_r}} \frac{1}{R_r^2} \times \\
 & \times \left\{ \left( \frac{T_r - T_\Delta}{T_r - T_\infty} + \ln \frac{R_\Delta}{R_r} \right) \left[ r^2 - R_r^2 - (R_\Delta^2 - R_r^2) \frac{\ln \frac{r}{R_r}}{\ln \frac{R_\Delta}{R_r}} \right] + \right. \\
 & \left. + (R_\Delta^2 - r^2) \ln \frac{r}{R_r} \right\}, \quad T = T_r + (T_\Delta - T_r) \frac{\ln \frac{r}{R_r}}{\ln \frac{R_\Delta}{R_r}}.
 \end{aligned} \quad (4)$$

Knowing the velocity and temperature distribution in the cell, the friction and heat flux at the rod may be determined and related to  $f$  and  $q$ :

$$\begin{aligned}
 f = \frac{2\pi R_r}{\rho \pi R_\Delta^2} \mu \frac{\partial u}{\partial r} \Big|_{r=R_r} & = \frac{2\nu(U_\Delta - U_r)}{R_\Delta^2 \ln \frac{R_\Delta}{R_r}} + \delta g \frac{T_r - T_\infty}{T_\infty \ln \frac{R_\Delta}{R_r}} \times \\
 & \times \left[ (\varepsilon + a_\varepsilon - 1) \left( \frac{T_r - T_\Delta}{T_\Delta - T_\infty} + \ln \frac{R_\Delta}{R_r} \right) - \frac{T_r - T_\Delta}{T_r - T_\infty} \frac{\varepsilon}{2} \right], \\
 q = -\frac{2\pi R_r}{\pi R_\Delta^2} \lambda \frac{\partial T}{\partial r} \Big|_{r=R_r} & = \frac{2\lambda(T_r - T_\Delta)}{R_\Delta^2 \ln \frac{R_\Delta}{R_r}}, \quad a_\varepsilon = \frac{\varepsilon}{2 \ln \frac{R_\Delta}{R_r}}.
 \end{aligned} \quad (5)$$

The arbitrary quantities  $U_\Delta$  and  $T_\Delta$  may be found on the assumption that the filtrational velocity is equal to the mean flow rate in the cell, and the temperature is equal to the mean calorimetric quantity of the flow in the cell

$$u_1 = \frac{2\pi}{\pi R_\Delta^2} \int_{R_r}^{R_\Delta} u r dr, \quad T_1 = \frac{2\pi}{\pi R_\Delta^2 u_1} \int_{R_r}^{R_\Delta} u T r dr. \quad (6)$$

Substitution of Eq. (4) into Eq. (6) gives, after appropriate manipulations

$$U_\Delta = U_r + Au + Bu(T_\Delta - T_r), \quad (7)$$

$$T_\Delta = T_r + \frac{1}{2At} [-B\tau + \sqrt{B\tau^2 - 4Atu_1(T_r - T_\infty)}],$$

where

$$Au = \frac{u_1 - \varepsilon U_r - \delta(Gr_r/Re_r) U_r c_1}{1 - a_\varepsilon};$$

$$Bu = \delta U_r (Gr_r/Re_r) \frac{c_1 + c_2}{(T_r - T_\infty)(1 - a_\varepsilon) \ln \frac{R_\Delta}{R_r}};$$

$$A\tau = \delta \frac{U_r (Gr_r/Re_r)}{(T_r - T_\infty) \ln \frac{R_\Delta}{R_r}} \left[ c_3 + c_5 + \frac{c_4(c_1 + c_2)}{1 - a_\varepsilon} \right];$$

$$B\tau = U_r(1 - a_\varepsilon) - \delta(Gr_r/Re_r) U_r c_5 + Au c_4.$$

The factors  $c_j$  depend only on the geometric parameters of the bundle

$$\begin{aligned}
 c_1 &= \frac{\varepsilon}{4(1-\varepsilon)} (1 - 0,5\varepsilon - a_\varepsilon), \\
 c_2 &= \frac{\ln \frac{R_\Delta}{R_r}}{8(1-\varepsilon)} (a_\varepsilon + 0,5\varepsilon a_\varepsilon - 1), \\
 c_3 &= \frac{1}{8(1-\varepsilon)} \left[ \ln \frac{R_\Delta}{R_r} + 1,5(a_\varepsilon - 1) + 0,25\varepsilon a_\varepsilon \right], \\
 c_4 &= \frac{1}{\ln \frac{R_\Delta}{R_r}} \left( \ln \frac{R_\Delta}{R_r} - 1 + a_\varepsilon \right), \\
 c_5 &= \frac{1}{8(1-\varepsilon)} [(1 - a_\varepsilon)(2\varepsilon - 1) + 0,5\varepsilon a_\varepsilon - 2\varepsilon c_4].
 \end{aligned} \tag{8}$$

By means of Eq. (7), local characteristics of the flow in the vicinity of the cylinder may be related to filtrational parameters at the given point, which are related to the whole flow field in the rod system and the surrounding medium by means of the filtrational equations and boundary conditions. It was shown in [7] that the solutions obtained in the cell for the friction and heat flux in induced convection are in good agreement with the known approximate and accurate numerical solutions for triangular and rectangular regular rod assemblies.

Thus, Eqs. (1)-(3), (5), and (7) completely define the problem of mixed convection in a system of moving rods in open space if the rod temperature is known. In calculating the cooling of heated cylinders, another equation must be added to this system. Assuming that the moving cylinder is thin and that the Biot number is considerably less than one, the heat-transfer equation of the rod is written in the form

$$\rho_r c_p U_r \frac{dT_r}{dx} = \frac{2\pi R_r}{\pi R_r^2} q_r, \quad q_r = -\lambda \left. \frac{\partial T}{\partial r} \right|_{r=R_r}; \tag{9}$$

this equation closes the heat-transfer problem for a moving system of cooled cylinders.

**Results and Analysis.** The above conjugate problem is solved numerically using the method of [12]. Two versions of mixed convection in the bundle are considered: convection in the direction of bundle motion and in the opposite direction. The distribution of filtrational velocities at the surface and in the center of the bundle over the length of the cooling zone is shown in Fig. 2. The basic parameters of the bundle are as follows:  $R_b = 0.02$  m;  $R_r = 0.42 \cdot 10^{-4}$  m;  $U_r = 0.5$  m/sec;  $T_{or} = 100^\circ\text{C}$ ;  $T_\infty = 20^\circ\text{C}$ ;  $N = 200$ ;  $Re_b = 667.0$ ;  $Gr_b = 9.52 \cdot 10^4$ . The thermophysical parameters of the surrounding medium correspond to the parameters of air.

The examples considered indicate the following features of filtrational flow. Whereas for induced convection the limiting filtration rate in the bundle is the velocity of fiber motion (it is evident from Fig. 2 that this velocity is attained when  $x = 0.3$  m), for convection in the direction of the bundle the velocity at the center of the bundle increases considerably more intensively and becomes greater than the velocity of rod motion. This is also observed at the bundle surface; because of the strong inflow of the medium, it is less than the filtrational velocity with induced convection at first, but with increase in  $x$  it increases, exceeding  $U_c$  and approaching the velocity at the center of the bundle. This behavior of the cooling medium is due to the forces acting in the direction of fiber motion associated with free convection in the heated volume of the bundle. With cooling of the fibers, the influence of the upward forces decreases; therefore, the filtrational velocity reaches a maximum, and then must approach  $U_c$  asymptotically with increase in  $x$ .

For convection in the opposite direction to the bundle, the viscous friction at the fiber surface is compensated to some extent by the influence of free convection. This leads to considerable reduction in filtrational velocity in the bundle (Fig. 2a).

The distribution of the filtrational velocities at the center and surface of the bundle over the length is shown in Fig. 2b for a less dense bundle ( $N = 100$ ) with convection in

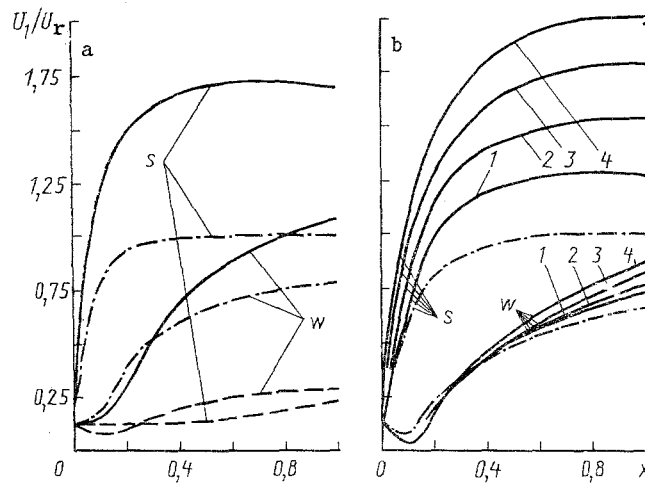


Fig. 2. Distribution of dimensionless filtrational velocities in mixed and induced convection in bundle (a) for  $\gamma = 0, \pm 142.8$  and  $N = 200$  and in forward convection (b) for  $N = 100$  and  $\gamma = 35.7$  (1), 71.4 (2), 107.1 (3), and 142.8 (4); s) velocity at center of bundle; w) at surface; continuous curves) forward convection; dash-dot curves) induced convection; dashed curves) inverse convection.  $x$ , meters.

the direction of the bundle motion and various values of  $\gamma = Gr_b/Re_b$ . Increase in  $\gamma$  leads to monotonic increase in the filtrational velocity in the bundle volume; for a less dense bundle, the velocity maximum is higher, other conditions being equal (curve 4 in Fig. 2b and continuous curve s in Fig. 2a). In addition, significant acceleration of the gas at the center in the initial section is associated with intense ejection of the cooling medium and considerable drop in velocity at the bundle surface. At some value of  $\gamma$ , the longitudinal velocity at the boundary may be considerably less than the transverse velocity, which prevents the use of the boundary-layer approach. Note that, at such values of  $\gamma$ , a region of return gas flow appears in the bundle for inverse convection.

These features of the hydrodynamic flow in the rod system also depend on the rate of cooling. The distribution of the fiber temperature at the surface and center of the bundle over the length is shown in Fig. 3 for  $N = 200$ . For comparison, the distribution of the temperature of single fibers moving at the same velocity with forward and inverse convection is also shown (continuous and dashed curves 1). It is evident that fiber cooling in a bundle depends significantly on the intensity of the hydrodynamic processes determined by the interaction of the bundle with the external medium. Thus, for the given case, the fiber temperature in the bundle differs fairly significantly from the temperature of the individual fibers

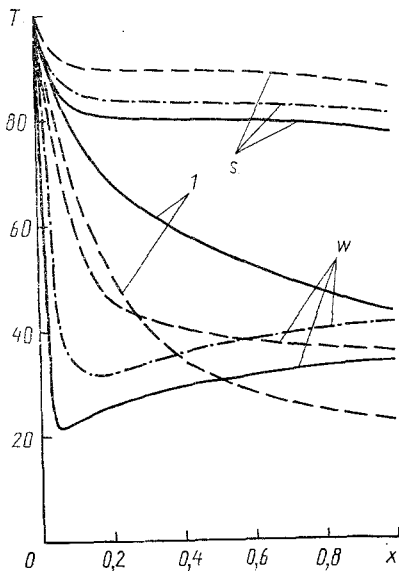


Fig. 3. Distribution of rod temperature on cooling in bundle and temperature of single rods in induced and mixed convection when  $N = 200$ ,  $\gamma = 0, \pm 142.8$ : 1) temperature of single fibers; s) temperature at center of bundle; w) at surface; continuous curves) forward convection; dash-dot curves) induced convection; dashed curves) inverse convection.  $T$ ,  $^{\circ}C$ .

( $R_r = 0.42 \cdot 10^{-4}$  m;  $Re_r = 1.4$ ;  $Gr_r = 0.0012$ ). The upper group of curves characterizes the temperature distribution of fibers at the center of the bundle. It is evident that they vary fairly weakly over the length; the curve for forward convection is somewhat lower than the others here. This indicates a higher rate of cooling than with inverse and induced convection, since in this case the bundle ejects a greater quantity of cooling medium. For the same reason, the fiber temperature at the bundle surface for forward convection varies fairly sharply, almost reaching the temperature of the external medium in a short cooling section. Further stabilization of the flow in the bundle leads to decrease in ejection and growth of the external boundary layer. Consequently, heat fluxes from the hotter inner layers of gas propagate to the external regions; the gas temperature in the surface layers begins to increase and, with it, the fiber temperature.

For this case, the fiber temperature with induced convection varies analogously. For inverse convection, the temperature dependence is monotonic in the given cooling section.

The heat-transfer intensity of the rods in the bundle may be characterized by the local Nusselt number:  $Nu_r = \alpha D_r / \lambda$ . Using Eq. (4), it is found that

$$Nu_r = - \frac{2}{\ln \frac{R_\Delta}{R_r}} \frac{T_\Delta - T_r}{T_r - T_1} \quad (10)$$

For induced convection, it is simple to express  $T_\Delta$  in terms of  $T_r$ ,  $T_1$ , and  $u_1$ , and  $Nu_r$  takes the form

$$Nu_r = 2 \ln^{-1} \frac{R_\Delta}{R_r} \left[ c_0 + (1 - a_\varepsilon - \varepsilon c_0) \frac{U_r}{u_1} \right]^{-1}, \quad (11)$$

where  $c_0 = c_4 / (1 - a_\varepsilon)$ .

It is evident that Eq. (11) is simplified with induced convection in a system of motionless rods ( $U_r = 0$ )

$$Nu_r^0 = 2c_0^{-1} \ln^{-1} \frac{R_\Delta}{R_r} = - \frac{2 [\ln(1 - \varepsilon) + \varepsilon]}{\ln(1 - \varepsilon)[0.5 \ln(1 - \varepsilon) + 1] + \varepsilon}, \quad (12)$$

i.e.,  $Nu_r^0$  depends only on the bundle porosity or on the relative packing step  $b/D_r$  for regular structures. In the case of heat transfer in a system of moving rods,  $Nu_r$  depends parametrically on the dynamic characteristic of the filtrational flow  $U_r/u_1$ , and must be determined in solving the complete conjugate problem in Eqs. (1)-(3) and (9). However, taking into account that dynamic stabilization sets in under induced convection in a system of moving rods in the basic volume of the bundle, except for a surface zone, i.e.,  $u_1 \rightarrow U_r$ , the following expression may be written for  $Nu_r$

$$Nu_r^1 = 2 \ln^{-1} \frac{R_\Delta}{R_r} [c_0 + (1 - a_\varepsilon - c_0 \varepsilon)]^{-1}.$$

In terms of the porosity

$$Nu_r^1 = - \frac{2 [\ln(1 - \varepsilon) + \varepsilon]}{0.5 [\ln(1 - \varepsilon) + \varepsilon]^2 + [0.5 \ln^2(1 - \varepsilon) + \ln(1 - \varepsilon) + \varepsilon](1 - \varepsilon)}. \quad (13)$$

In the general case of unstabilized mixed convection,  $T_\Delta$  must be calculated from Eq. (7) in determining  $Nu_r$ . The fairly complex dependence  $T_\Delta(u_1, T_1)$  does not yield simple analytical expressions for  $Nu_r$ . Some simplification of the expression for  $Nu_r$  is possible under the condition of thermal stabilization in the bundle  $T_1 \rightarrow T_r$ , permitting linearization of the relation for  $T_\Delta$ . As a result

$$Nu_r = 2 [A + B(U_r/u_1) - \delta D (Gr_r/Re_r)(U_r/u_1)]^{-1}, \quad (14)$$

$$A = c_0 \ln \frac{R_\Delta}{R_r}, \quad B = (1 - a_\varepsilon - \varepsilon c_0) \ln \frac{R_\Delta}{R_r},$$

$$D = (c_5 + c_0 c_1) \ln \frac{R_\Delta}{R_r}.$$

TABLE 1. Dependence of A, B, and D in Eq. (14) on Bundle Porosity  $\epsilon$

$\epsilon$	0,5	0,6	0,7	0,8	0,9	0,95
A	0,243	0,327	0,438	0,6	0,89	1,19
B	-0,025	-0,038	-0,055	-0,075	-0,099	-0,11
D	0,0004	0,0012	0,0036	0,0118	0,0533	0,1728

The parametric dependence in Eq. (14) enables the influence of various factors on the heat transfer in the bundle to be sufficiently clearly shown. The first term in the denominator depends only on the packing density of the bundle or  $\epsilon$ ; the second term takes account of the contribution of dynamic flow parameters; and the third expresses the influence of free convection. Table 1 gives A, B, D as a function of the bundle porosity. Taking into account that  $B \sim 10^{-1}A$ ,  $D \sim 10^{-1}A$  approximately, and that  $Gr_r/Re_r \sim 10^{-2}$  for most fiber bundles, it may be concluded that  $Nu_r$  is practically independent of  $Gr_r/Re_r$  for stabilized forward mixed convection, is determined solely by the packing density of the bundle, and may be calculated on the basis of Eq. (13). For intense dynamic flows in bundles, when  $u_1$  is considerably greater than  $U_r$ , approximate values of  $Nu_r$  may be obtained from Eq. (12).

The dependence of  $Nu_r^0$  and  $Nu_r^1$  on the porosity  $\epsilon$  is shown in Fig. 4, together with the dependence of  $Nu_r$  for the case of stabilized convection in an infinite rod assembly (curve 2 and the data of approximate solution [13] and accurate numerical solution [14]). These results suggest that, for stabilized convection in rod bundles, despite the different heat-transfer conditions (gradient flow, nongradient flow, induced flow, mixed-convective flow), there is some dependence  $Nu_r(\epsilon)$  which approximates the data in the range  $0.6 \leq \epsilon < 1$  to within  $\pm 10\%$  (is close to 1 in cases that are of practical importance). For example, Eq. (13) may be such a dependence.

For inverse convection in bundles of moving fibers, when there is no dynamic stabilization, the estimates in Eqs. (12) and (13) may give large errors. Thus, curve 4 in Fig. 4 is obtained for  $U_r/u_1 = 3.3$ . This difference arises in that in the given case the terms A and  $B(U_r/u_1)$  become commensurate in value, i.e., the influence of dynamic parameters on the heat transfer of the rod increases significantly. The value of  $Nu_r$  may then only be obtained from the solution of the complete problem, and Eq. (13) may be used to estimate the lower limit on  $Nu_r$ .

Theoretical and experimental data on free convection in regular rod assemblies were given in [15]. For the relative packing step  $b/D_r = 1.68$ , the experimental range of Nusselt numbers observed is  $Nu_r = 4.7-5.4$ ; for  $b/D_r = 2.03$ ,  $Nu_r = 3.8-4.2$ . The corresponding values

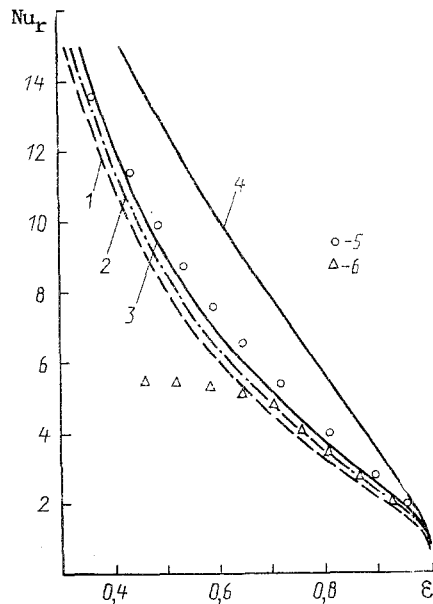


Fig. 4. Dependence of local Nusselt number  $Nu_r$  in rod bundle on porosity  $\epsilon$ : 1) Eq. (12); 2) dependence of [7]; 3) Eq. (13); 4) Eq. (11) with  $U_r/u_1 = 3.3$ ; 5) data of [13]; 6) data of [14].

given by Eq. (13) are:  $Nu_r = 5.55$  and  $Nu_r = 4.08$ . The good agreement of the experimental and theoretical values of  $Nu_r$  suggests that the heat-transfer model proposed is adequate and the dependences  $Nu_r^0(\epsilon)$ ,  $Nu_r^1(\epsilon)$  are reliable.

The given mathematical model of the motion and heat transfer in a bundle of moving rods may be used in design practice to estimate the efficiency of various methods of cooling fiber bundles and to choose the basic design parameters of technological devices. In addition, the present results may serve as the basis for the development of a more general model of synthetic-fiber and glass-fiber formation in bundles, and the dependences of the local Nusselt numbers may be used to create engineering methods of thermal calculation of such processes.

#### NOTATION

$x$ ,  $r$ , longitudinal and radial coordinates of cylindrical system;  $u$ ,  $v$ , corresponding velocity components;  $T$ , temperature;  $\rho$ , density;  $c_p$ , specific heat at constant pressure;  $\lambda$ , thermal conductivity;  $\nu$ , kinematic viscosity;  $\mu$ , dynamic viscosity;  $U_r$ ,  $T_r$ , rod velocity and temperature;  $T_\infty$ , temperature of surrounding medium;  $T_{0C}$ , initial rod temperature;  $D_r$ , rod diameter;  $R_r$ , rod radius;  $R_\Delta$ , extremal radius of cell;  $R_b$ , radius of rod bundle;  $\epsilon$ , porosity;  $Re_r = U_r R_r / \nu$ , Reynolds number of rod;  $Re_b = U_r R_b / \nu$ , Reynolds number of bundle;  $Gr_r = g R_r^3 \times (T_c - T_\infty) / (\nu^2 T_\infty)$ , Grashof number of rod;  $Gr_b = g R_b^3 (T_r - T_\infty) / (\nu^2 T_\infty)$ , Grashof number of bundle;  $f$ ,  $q$ , volume resistance and power of heat sources;  $N$ , number of rods in bundle;  $\gamma = Gr_b / Re_b$ ;  $Nu_r = \alpha D_r / \lambda$ , Nusselt number for rod;  $b/D_r$ , relative packing step of rods in bundle. Indices: 1, filtrational parameters of gas in bundle; 2, parameters of gas in external spatial layer;  $r$ , rod parameters;  $b$ , bundle parameters.

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